CyberChallenge.IT 2020 - Pretest
Commented solutions

Contents

1 Question 1

1.1 Question

Consider a two-pan balance scale.
There are 32 tennis balls. 31 of them have the same weight and the thirty-second is a bit heavier.
In the worst case scenario, what is the minimum number of weighings you need to identify the heavy one?

1.2 Answers

(A) 3

(B) 4

(C) 5

(D) 6

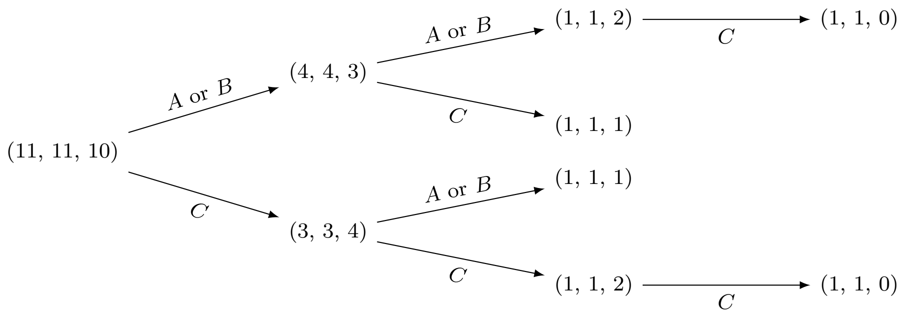
1.3 Proposed solution

The correct answer is `(B) 4`.

The optimal strategy is to divide the balls into 3 groups, let us say A, B, C, and compare two of them with the balance scale. Without loss of generality, we compare the weights of A and B. If one of them is heavier than the other, we know that the ball we have to find is in that group. Otherwise, if the two groups have the same weight, the ball is in group C.

Since 32 is not divisible by 3, we cannot split the balls evenly, but one or two groups will result slightly larger the the other(s).

The global strategy can be represented by the following tree, where the triplets correspond to the division in the groups A, B, C and the edge labels shows the heaviest group.



When a node with every group not larger than 1 is reached, the search stops since we can determine the heaviest ball with that weighing. We are looking for the worst case scenario, so the correct answer is the maximum number of weighings in this tree, which is 4.

2 Question 2

2.1 Question

Only one of the following sentences is true.

- Alice: "Bob is the tallest"
- Bob: "Charlie is the tallest"
- Charlie: "I'm not the tallest"
- David: "I'm not the tallest"

Who is the tallest?

2.2 Answers

(A) David

(B) Charlie

(C) Alice

(D) Bob

2.3 Proposed solution

The correct answer is `(A) David`.

If only one sentence must be true, than it should be one of the two said by Charlie and David, since one of the two must not be the tallest one. If David's sentence is true, then Charlie lies and he is the tallest one, thus also Bob said the truth, which is a contraddiction.

The conclusion is that David must be the tallest.

3 Question 3

3.1 Question

Given 100 balls in a box, numbered from 1 to 100, what is the probability of extracting a perfect cube number?

3.2 Answers

(A) 1/25

(B) 1/20

(C) 1/24

(D) 4/50

3.3 Proposed solution

The correct answer is `(A) 1/25`.

The perfect cubes lower than 100 are:

- $1^3 = 1$
- $2^3 = 8$
- $3^3 = 27$
- $4^3 = 64$

The next cube would be $5^3 = 125 > 100$.

Thus, there are 4 balls with a perfect cube on them and a total of 100 balls, so the probability is $\frac{4}{100} = \frac{1}{25}$.

4 Question 4

4.1 Question

Consider 3 different programmers:

- Alice can finish a project in 1 hour
- Bob can finish a project in 2 hours
- Charlie can finish a project in 6 hours

How long would they take to complete a project?

4.2 Answers

(A) 30 minutes

(B) 36 minutes

(C) 42 minutes

(D) 24 minutes

4.3 Proposed solution

The correct answer is `(B) 36 minutes`.

Let us compute the programmers' speed:

- Alice works at 1 project/hour
- Bob works at $\frac{1}{2}$ projects/hour
- Charlie works at $\frac{1}{6}$ projects/hour

The sum of the speeds is $s = 1 + \frac{1}{2} + \frac{1}{6} = \frac{10}{6} = \frac{5}{3}$ projects/hour.

In order to complete one project, $1/s = \frac{3}{5}$ hours are needed, which corresponds to 36 minutes.

5 Question 5

5.1 Question

Given the even numbers between 2 and 2020, how many numbers are not divisible by 5 or 7?

5.2 Answers

(A) 808

(B) 692

(C) 866

(D) 1384

5.3 Proposed solution

The correct answer is `(B) 692`.

Given a number n, the natural numbers up to the number m that are divisible by n are $\left\lfloor \frac{m}{n} \right\rfloor$.

In our case, we have $\left\lfloor \frac{2020}{2} \right\rfloor = 1010$ even numbers, $\left\lfloor \frac{2020}{2 \cdot 5} \right\rfloor = 202$ even numbers divisible by 5 (i.e. they are divisible by 10) and $\left\lfloor \frac{2020}{2 \cdot 7} \right\rfloor = 144$ even numbers divisible by 7 (i.e. divisible by 14).

But if we subtract from 1010 these two last values, we are subtracting the even numbers divisible by both 5 and 7 two times. Therefore, we have to add again $\left\lfloor \frac{2020}{2 \cdot 5 \cdot 7} \right\rfloor = 28$, which is how many numbers are divisible by 70. The result is $1010 - 202 - 144 + 28 = 692$.

6 Question 6

6.1 Question

Infinite coins of 1, 2 and 5 cents are available. How many different ways we can make 7 cents?

6.2 Answers

(A) 1

(B) 4

(C) 6

(D) 10

6.3 Proposed solution

The correct answer is `(C) 6`.

7 cents can be obtained from 1, 2 and 5 cents coins in the following ways:

- $1 + 1 + 1 + 1 + 1 + 1 + 1$
- $1 + 1 + 1 + 1 + 1 + 2$
- $1 + 1 + 1 + 2 + 2$
- $1 + 1 + 5$
- $1 + 2 + 2 + 2$
- $2 + 5$

for a total of 6 possibilities.

7 Question 7

7.1 Question

Consider a fruit basket.

- All fruits except 3 are apples
- All fruits except 4 are oranges
- All fruits except 1 are bananas

How many fruits are there in total?

7.2 Answers

(A) 6

(B) 12

(C) 4

(D) 8

7.3 Proposed solution

The correct answer is `(C) 4`.

Let us call wit a, b, o the numbers of apples, bananas and oranges, respectively. The question can be summarized as

$$\begin{cases} b + o = 3 \\ a + b = 4 \\ a + o = 1 \end{cases}$$

The solution of the system above is

$$\begin{cases} a = 1 \\ b = 3 \\ o = 0 \end{cases}$$

for a total of 4 fruits.

8 Question 8

8.1 Question

John has 1260 candies. He wants to give them in equal number to each of his friends, that are less than 100. How many friends has John at most?

8.2 Answers

(A) 70

(B) 84

(C) 90

(D) 94

8.3 Proposed solution

The correct answer is `(C) 90`.

We can either factor $1260 = 2^2 \cdot 3^3 \cdot 5 \cdot 7$ and list its divisors $(\ldots, 84, 90, 105, 126, \ldots)$ or we can just check, among the proposed answers, the highest number that divides 1260: 94 does not divide 1260, but 90 does.

9 Question 9

9.1 Question

What is the greatest positive integer n such that the arithmetic mean of the numbers from 1 to n is less than 2020?

9.2 Answers

(A) 4038

(B) 4039

(C) 4040

(D) 2020

9.3 Proposed solution

The correct answer is `(A) 4038`.

The arithmetic mean of the numbers from 1 to n can be expressed as $AM = \frac{\sum_{i=1}^{n} i}{n} = \frac{n \cdot (n+1)}{2n} = \frac{n+1}{2}$. Adding the constraint that it should be less than 2020, we have $AM < 2020; \frac{n+1}{2} < 2020; n < 4039$ and the greatest integer that respects this contraint is 4038.

10 Question 10

10.1 Question

Consider the following incomplete function which sorts three numbers

```
function sort3(a, b, c) {
    if($1) {
        if($2) {
            print(a, b, c)
        } else {
            if($3) {
                print(a, c, b)
            } else {
                print(c, a, b)
            }
        }
    } else {
        if($4) {
            if($5) {
                print(b, a, c)
            } else {
                print(b, c, a)
            }
        } else {
            print(c, b, a)
        }
    }
}
```

which of the following conditions for `$1`, `$2`, `$3`, `$4` and `$5` makes the program correct?

10.2 Answers

(A) `(a < b), (a < c), (b < c), (c > a), (b < c)`

(B) `(a < b), (a < c), (b > c), (a < c), (b < c)`

(C) `(a < b), (b < c), (a < c), (b < c), (a < c)`

(D) `(a < b), (b < c), (a < c), (a < c), (b < c)`

10.3 Proposed solution

The correct answer is `(C) (a < b), (b < c), (a < c), (b < c), (a < c)`.

Analyzing all the results inside the first `if`, we notice that `a` is always before `b` (and vice-versa in the `else`), so `$1` must be `a < b` (assuming that the function sorts in ascending order or looking at the answers). Analogously, in the second `if` we have `b` before `c` (and vice-versa in the `else`), so `$2` must be `b < c`. Finally, the fourth `if` discriminates between `b` and `c`, so `$4` must be `b < c` and this is sufficient to identify the correct answer.

11 Question 11

11.1 Question

Consider the following function that accept an unsigned integer number greater than two

```
function mistery(n) {
    k = 2
    while((k*k <= n) and (n mod k > 0)) {
        k = k+1
    }
    return (k*k > n)
}
```

What does the function return?

11.2 Answers

(A) false if `k < n`, true otherwise

(B) false if `n` is prime, true otherwise

(C) true if `n=k*k`, false otherwise

(D) true if `n` is prime, false otherwise

11.3 Proposed solution

The correct answer is `(D) true if n is prime, false otherwise`.

The function is checking that, for all k such that $k^2 \leq n$, $n \pmod{k} \neq 0$. This is a common algorithm to check if n is prime[1], since we iterates over the range that includes half of its divisors (if d is a divisor of n greater than $\sqrt{n}$, then $\frac{n}{d}$ is still a divisor of n, but less than $\sqrt{n}$).

At the end, `k*k > n` checks if we iterated over the entire range (so, no divisors are found, and thus n is prime) or, otherwise, a factor of n is found.

[1] See `https://en.wikipedia.org/wiki/Primality_test#Simple_methods`

12 Question 12

12.1 Question

Consider a function that checks if a vector V of size N is palindrome

```
function check(V[], N){
    result = true
    for(i=0; i<$1; i++) {
        if($2) {
            result = false
        }
    }
    return result
}
```

Which of the following code fragments for `$1` and `$2` makes the program more correct?

12.2 Answers

(A) `N and (V[i] != V[N-i])`

(B) `N/2 and (V[i] != V[N-i])`

(C) `N-1 and (V[i] != V[N-i-1])`

(D) `N/2 and (V[i] != V[N-i-1])`

12.3 Proposed solution

The correct answer is `(D) N/2 and (V[i] != V[N-i-1])`.

In order to check if a vector is palindrome, we have to compare the first element with the last one, the second with the second to last, and so on. Since in C vectors are 0-indexed, the valid values to be used as indexes for a vector of size `N` are `0, 1, ...N-2, N-1`. It follows that the correct fragment that replaces `$2` is `V[i] != V[N-i-1]`.

Both answers `(C)` and `(D)` are valid, but iterating from `0` to `N-1` excluded forces the function to check almost all the couples twice (except for the most external one), while stopping at `N/2` is optimal (every couple is checked only once).